

Peculiarities of the stochastic motion in antiferromagnetic nanoparticles

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Antiferromagnetic (AFM) materials are widely used in spintronic devices as passive elements (for stabilization of ferromagnetic layers) and as active elements (for information coding). In both cases switching between the different AFM states depends in a great extent from the environmental noise. In the present paper we derive the stochastic Langevin equations for an AFM vector and corresponding Fokker-Planck equation for distribution function in the phase space of generalised coordinate and momentum. Thermal noise is modeled by a random delta-correlated magnetic field that interacts with the dynamic magnetisation of AFM particle. We analyse in details a particular case of the collinear compensated AFM in the presence of spin-polarised current. The energy distribution function for normal modes in the vicinity of two equilibrium states (static and stationary) in sub- and super-critical regimes is found. It is shown that the noise-induced dynamics of AFM vector has peculiarities compared to that of magnetisation vector in ferromagnets.

I. INTRODUCTION

Magnetic nanoparticles are the main constituents of the nowadays devices for information technology. While the deterministic dynamics of magnetisation vectors is used for information coding, the noise-induced stochastic behaviour facilitates the switching processes and thus is used to increase the speed of information processing (see, e.g.^{1,2}). Noise measurement is a powerful and informative tool for study of the spintronic effects in different systems^{3,4}. The details of the stochastic processes are also important for the development of high-quality spin-torque oscillators⁵ and micropower generators^{6,7}.

Theoretical approach to the description of thermal noise in small *ferromagnetic* (FM) particles was developed in the seminal papers of W.F. Brown^{8,9} where the thermal bath was modeled by fluctuating magnetic field and corresponding Langevin equations were obtained as generalisation of the dynamic Landau-Lifshitz-Gilbert equations. This approach was then extended to the systems in the vicinity of the Curie point¹⁰, space-inhomogeneous magnetic vortices¹¹, systems with the coloured noise^{12,13} and FMs in the presence of spin-polarised current^{14,15}.

On the other hand, stochastic behaviour of *antiferromagnetic* (AFM) nanoparticles which are also widely used in electronic and spintronic devices is studied to a much lesser extent. The main reason is in the seeming magnetic neutrality of AFMs which manifests itself in the vanishingly small or zero net magnetisation, quadratic (in contrast to linear for FMs) dependence of the internal energy *vs* external magnetic field etc. Thus, up to now the problem of magnetic relaxation and thermal noise in AFM particles was in fact reduced to the description of the effective FM particle with small but nonzero magnetisation which inevitably appears due to imperfections, strong external magnetic fields, surface effects etc^{16–19}. However, in many cases a peculiar feature of AFM, namely, the presence of strong exchange coupling between the differently aligned (mainly in opposite directions) magnetic moments, gives rise to new dynamic effects that could not be reduced to the motion of the above mentioned “stray” magnetisation. For example, in contrast to magnetisation of FM particle, an AFM (Néel) vector (defined as the difference between sublattice magnetisations) can be set into motion not only by external magnetic field \mathbf{H} but also by its time derivative $\dot{\mathbf{H}}$ (or curled electric field)²⁰. Typical frequencies of AFM oscillations are fall into terahertz range (compared to 1-10 GHz for FM) due to effects of the exchange enhancement. AFM systems, like FMs, are sensitive to spin-torques transferred from the spin-polarised current^{21–23} but the current-induced AFM dynamics differs significantly from the current-induced dynamics of FMs²⁴. Thus, with the account of perspective to use AFM nanoparticles as alternative to FM active elements of spin-valves (see, e.g.²⁵), theoretical study of thermal noise in these systems is of importance and of great interest.

In the present paper we generalise the dynamic equations for AFM nanoparticle to the stochastic Langevin equations that describe “Brownian motion” of AFM vector in the presence of thermal noise. Like in the W. F. Brown approach we model the noise as a fluctuating delta-correlated magnetic field which interacts with the *dynamic* (induced by the motion of AFM vector^{26,27}) magnetisation of AFM. Using the standard Langevin dynamics technique for multiplicative noise²⁸ we derive the Fokker-Planck equations for the distribution function in the phase space of AFM vector and corresponding generalised momentum and discuss the peculiarities of AFM system compared to FM one. As a representative example we analyse the stochastic behaviour of AFM particle in the presence of spin-polarised current and find the energy distribution function in subcritical and supercritical regimes. For the sake of simplicity we restrict ourselves with the case of a collinear compensated AFM with two oppositely directed magnetic sublattices. However, the developed approach allows generalisation for the multisublattice AFM, weak ferromagnets etc.

II. LANGEVIN EQUATIONS FOR ANTIFERROMAGNETIC PARTICLE

In what follows we consider the fine (nanosized) magnetic particles whose state can be described with only few macroscopic vectors²⁹ \mathbf{M}_j : magnetisation vector ($j = 1$) in the case of FM nanoparticle and vectors of two equivalent sublattice magnetizations ($j = 1, 2$) in the case of a collinear AFM.

Let us first discuss the stochastic Landau-Lifshitz-Gilbert equation for the FMs which describes the dynamics of magnetisation \mathbf{M} subjected to a spin transfer torque \mathbf{T}_{STT} at a finite temperature T ¹⁴:

$$\dot{\mathbf{M}} = \gamma \mathbf{M} \times (\mathbf{H}_{\text{eff}} + \mathbf{h}) + \frac{\gamma \alpha_G}{M} \mathbf{M} \times [\mathbf{M} \times (\mathbf{H}_{\text{eff}} + \mathbf{h})] + \mathbf{T}_{\text{STT}}, \quad (1)$$

where γ is the gyromagnetic ratio, $\mathbf{H}_{\text{eff}} \equiv -\partial w_{\text{an}}(\mathbf{M})/\partial \mathbf{M}$ is the effective (combination of internal and external) magnetic field, $w_{\text{an}}(\mathbf{M})$ is the magnetic anisotropy energy with Zeeman contribution, α_G is the dimensionless damping (Gilbert) constant, the sign \times means cross-product. The fluctuating magnetic field $\mathbf{h}(t)$ with a Gaussian stochastic process has the following standard space-time statistical properties:

$$\langle \mathbf{h}(t) \rangle = 0, \quad \langle h_j(t_1) h_k(t_2) \rangle = 2D \delta_{jk} \delta(t_1 - t_2), \quad (2)$$

where D represents the strength of thermal fluctuations whose value is defined from the fluctuation-dissipation theorem. Symbol $\langle \dots \rangle$ denotes an average taken over all realizations of fluctuating field.

The first r.h.s. term in Eq. (1) describes nondissipative dynamics of magnetisation which is analogue of the precession of gyroscope with the “angular momentum” \mathbf{M} . The second r.h.s. term describes dissipation processes that have the same origin as fluctuating field $\mathbf{h}(t)$. Both nondissipative and dissipative terms are subjected to noise, however, the nondissipative noise is additive while the dissipative one is multiplicative³⁰. Delta-correlation of fluctuating field (2) means that the noise correlation time is much lesser than the characteristic time of magnetisation response.

The dynamics and kinetics of FM magnetisation with account of normalisation condition $|\mathbf{M}| = M_0$ (imposed far below the Curie point) is described with two independent variables that define space orientation of vector \mathbf{M} , in other words, by the variables of *configuration* space. Full “mechanical” energy of FM particle, $E_{\text{FM}} = w_{\text{an}}(\mathbf{M})$, also depends upon orientation of \mathbf{M} and thus can be treated as consisting of the potential energy only, in contrast to the energy of AFM particle (see below).

Using the analogy with FMs, one can derive the stochastic equation for AFMs from the corresponding dynamic equations assuming that the thermal noise also has the magnetic nature and can be modeled with the same random field $\mathbf{h}(t)$ (2). This field may originate from fluctuations of *i*) the surface noncompensated magnetisation for the small particles; *ii*) magnetisation of the nearest FM layer in spin-valves and multilayers; *iii*) current that produces additional magnetic field.

However, the deterministic dynamics in AFMs substantially differs from that of FM magnetisation and looks like an inertial motion of a point mass in a potential well. Formally this effect was demonstrated for AFMs with strong exchange coupling between the magnetic sublattices^{26,27}. In this case the dynamics of a collinear AFM is described by a single AFM (so called Néel) vector $\mathbf{L} \equiv \mathbf{M}_1 - \mathbf{M}_2$ of a fixed length ($|\mathbf{L}| = 2M_0$).

Deterministic equations of motion for \mathbf{L} could be obtained either from the set of Landau-Lifshits-Gilbert equations (1) for each of magnetic sublattices or, equivalently, in the framework of Lagrange formalism (see³¹ for details). The last approach is more convenient for general analysis, so, we start from the Lagrange function for AFM particle in the following form

$$\mathcal{L}_{\text{AFM}} = \frac{m_L}{2} \dot{\mathbf{L}}^2 + \gamma m_L [\dot{\mathbf{L}} \cdot (\mathbf{L} \times \mathbf{H})] - w_{\text{an}}(\mathbf{L}) + \frac{\gamma^2 m_L}{2} (\mathbf{L} \times \mathbf{H})^2, \quad (3)$$

where \mathbf{H} is an external magnetic field and $w_{\text{an}}(\mathbf{L})$ is the energy of magnetic anisotropy that forms a potential well for AFM vector, γ , as above, is the gyromagnetic ratio. The value $m_L \equiv 1/(2\gamma^2 M_0 H_E)$ plays a role of the “inertia mass”; it depends upon the spin-flip field of the exchange nature, $2H_E$, that characterises the intersublattice coupling.

In contrast to FMs, the nondissipative dynamics of AFMs can be also described within the Hamiltonian formalism with the following generalised energy (Hamilton function³²) obtained from (3):

$$\mathcal{H}_{\text{AFM}} = \frac{1}{2m_L} \mathbf{P}_L^2 - \gamma [\mathbf{P}_L \cdot (\mathbf{L} \times \mathbf{H})] + w_{\text{an}}(\mathbf{L}). \quad (4)$$

Here the generalised momentum $\mathbf{P}_L \equiv \partial \mathcal{L}_{\text{AFM}} / \partial \dot{\mathbf{L}}$ is canonically conjugated to the generalised coordinate \mathbf{L} .

As can be directly seen from (4), the generalised energy of AFM particle, \mathcal{H}_{AFM} , includes both kinetic (first term) and potential (last term) contributions. It means that the dynamics and kinetics of this system is described within the *phase* (*vs* configuration for FMs) space.

Dissipation is modeled with the Raileigh function which in the presence of spin-polarised current J takes a form²⁴:

$$\mathcal{R}_{\text{AFM}} = \gamma_{\text{AFM}} m_L \dot{\mathbf{L}}^2 - \frac{\sigma J}{2\gamma M_0} [\mathbf{p}_{\text{curr}} \cdot (\mathbf{L} \times \dot{\mathbf{L}})]. \quad (5)$$

Here the first term models the internal damping, damping coefficient $2\gamma_{\text{AFM}}$ is the AFMR linewidth, the constant $\sigma = \hbar\gamma\varepsilon/(2eM_0v_{\text{AFM}})$ is proportional to the efficiency ε of the spin transfer processes, v_{AFM} is the volume of AFM nanoparticle, \hbar is the Plank constant, e is the electron charge. Unit vector \mathbf{p}_{curr} is parallel to the direction of the current spin polarisation.

Thus, the stochastic equations for AFM in the phase space $\{\mathbf{L}, \mathbf{P}_L\}$ obtained from (3) and (5) with substitution $\mathbf{H} \rightarrow \mathbf{h}(t)$ acquire the form:

$$\begin{aligned} \dot{\mathbf{L}} &= \mathbf{P}_L / m_L - \gamma \mathbf{L} \times \mathbf{h} \\ \dot{\mathbf{P}}_L &= \mathbf{F}_L + \mathbf{F}_{\text{diss}} - \gamma (\mathbf{P}_L - 2\gamma_{\text{AFM}} m_L \mathbf{L}) \times \mathbf{h}, \end{aligned} \quad (6)$$

where $\mathbf{F}_L \equiv -\partial w_{\text{an}}(\mathbf{L}) / \partial \mathbf{L}$ is the potential (gradient) force, and the dissipative force \mathbf{F}_{diss} is given by the following expression

$$\mathbf{F}_{\text{diss}} \equiv - \left. \frac{\partial \mathcal{R}_{\text{AFM}}}{\partial \dot{\mathbf{L}}} \right|_{\dot{\mathbf{L}} \rightarrow \mathbf{P}_L} = -2\gamma_{\text{AFM}} \mathbf{P}_L - \frac{\sigma J}{2\gamma M_0} \mathbf{p}_{\text{curr}} \times \mathbf{L}. \quad (7)$$

Equations (6) describe the evolution of AFM vector in the presence of thermal noise and in this sense are analogous to the stochastic Eqs. (1) for magnetisation \mathbf{M} of FMs. Both sets of equations (for both FMs and AFMs) are linear in the random magnetic field \mathbf{h} . For an AFM system this fact is nonobvious and can be explained by the presence of small but nonzero macroscopic magnetisation $\mathbf{M}_{\text{AFM}} \equiv \mathbf{M}_1 + \mathbf{M}_2$ that in the compensated AFM has a dynamic origin²⁶ and can be expressed in terms of the Néel vector: $\mathbf{M}_{\text{AFM}} \propto \hat{\mathbf{L}} \times \mathbf{L} \propto \mathbf{P}_L \times \mathbf{L}$. On the other hand, the noise terms in both (FMs and AFMs) equations are *multiplicative* and this can, in principle, result in a possible stochastic resonance.

It should be stressed that though the AFM dynamics is similar to the dynamics of point mass, Eqs. (6) have one peculiarity compared with the standard Langevin equations for a Brownian particle in a potential well. Namely, the first of Eq. (6) includes the noise and does not include any dissipation term. This means that within the accepted model of dissipation (and noise) there is no time-scale separation between relaxation of generalised coordinate and generalised momentum. This fact is a direct consequence of limitations on $|\mathbf{L}|$ imposed by assumption of strong exchange coupling and absence of the exchange relaxation. However, the characteristic energy of exchange coupling is of the order of the Néel temperature. So, for low (compared with the Néel) temperatures and relatively small (compared with H_E) external fields Eqs. (6) give an adequate description of AFM vector behaviour.

The Langevin Eqs. (6) generate the Fokker-Planck equation for AFM probability distribution function $f(\mathbf{L}, \mathbf{P}_L; t)$ in the phase space:

$$\begin{aligned} \frac{\partial f}{\partial t} = & \gamma^2 \nabla_{\mathbf{L}} \cdot \left[\left(-2M_0 H_E \mathbf{P}_L + D \hat{\mathbf{L}} \cdot \nabla_{\mathbf{L}} - \mathbf{P}_L \otimes \mathbf{L} \cdot \nabla_{\mathbf{P}_L} \right) f \right] \\ & + \nabla_{\mathbf{P}_L} \cdot \left[\left(\mathbf{F}_L + \mathbf{F}_{\text{diss}} + D \gamma^2 \hat{\mathbf{P}}^L \cdot \nabla_{\mathbf{P}_L} - \mathbf{L} \otimes \mathbf{P}_L \cdot \nabla_{\mathbf{L}} \right) f \right], \end{aligned} \quad (8)$$

where we introduced the symbol $\hat{\mathbf{a}} \equiv \hat{\mathbf{a}}^2 - \mathbf{a} \otimes \mathbf{a}$ with $\mathbf{a} = \mathbf{L}$ or \mathbf{P}_L , and omitted small noise terms with γ_{AFM} for the sake of clarity.

Fokker-Planck Eq. (8) for AFM nanoparticles is, in fact, the main result of this paper. It is much more complicated than the analogous equations for FMs and (in contrast to FMs) could not be solved in general case even for stationary conditions. In the next section we consider some limiting cases that allow to find approximate stationary solutions, $f(\mathbf{L}, \mathbf{P}_L)$, and evaluate D from fluctuation-dissipation theorem.

III. ANTIFERROMAGNET PROBABILITY DISTRIBUTION IN THE PRESENCE OF SPIN-POLARISED CURRENT

The phase space of AFM particle is four-dimensional and this substantially complicates analysis of Eq. (8) in general case. However, in some cases the effective dimensionality can be reduced. The simplest case concerns the system in the vicinity of equilibrium where all the possible motions of AFM vector could be represented in terms of two noninteracting normal modes with the amplitudes c_{\pm} and angular phases φ_{\pm} . If, in addition, we neglect inhomogeneity in the phase φ_{\pm} distribution, then, distribution function can be factorized as: $f(\mathbf{L}, \mathbf{P}_L; t) = f_+(c_+; t) f_-(c_-; t)$. In what follows we consider the case of AFM with the degenerate excitation spectra for which two normal modes correspond to clockwise/counter-clockwise rotations of AFM vector around z axis with the frequency Ω_{AFMR} (that is close to AFMR frequency).

Spin current polarised along z axis ($\mathbf{p}_{\text{curr}} \parallel z$) interacts with both modes thus enhancing the effective damping of one (say, “+”) and diminishing the effective damping of the other (say, “-”)²⁴. In the subcritical regime ($|J| < J_{\text{crit}} \equiv 2\gamma_{\text{AFM}}\Omega_{\text{AFMR}}/(\gamma\sigma H_E)$, positive damping) the static equilibrium state is stable and normal modes are still well separated. In the supercritical regime, $|J| > J_{\text{crit}}$, an amplitude of one of the mode growth to saturation value and the stable state corresponds to rotation of AFM vector in xy plane with the current-dependent frequency $\omega = J\Omega_{\text{AFMR}}/J_{\text{crit}}$ ²⁴. Another normal mode corresponds to small oscillations of AFM vector in z direction, so, again, both modes are well separated. Thus, the behaviour of AFM vector in the subcritical and supercritical regions can be really described in approximation of two independent normal modes.

To obtain the Fokker-Planck equations for $f(c_{\pm})$ we use the approach of energy representation for nonequilibrium Brownian-like systems developed in³³. To this end let us start from the Langevin equation for the energy $E_{\text{AFM}} \equiv \mathbf{P}_L^2/(2m_L) + w_{\text{an}}(\mathbf{L})$ (compare with (4)):

$$\frac{dE_{\text{AFM}}}{dt} = -\mathbf{P}_L \cdot \mathbf{F}_{\text{diss}} + \gamma \left[\left(2\gamma_{\text{AFM}} \mathbf{P}_L - \frac{\partial w_{\text{an}}}{\partial \mathbf{L}} \right) \cdot \mathbf{L} \times \mathbf{h} \right]. \quad (9)$$

where the summands in the r.h.s. of Eq. (9) should be expressed in terms of E_{AFM} .

In approximation of noninteractive normal modes Eq. (9) is applicable to the energy $E_{\pm} = 4M_0^2\Omega_{\text{AFMR}}^2 m_L c_{\pm}^2$ of *each* mode. Moreover, within the accepted approximation (fixed oscillation frequency Ω_{AFMR}) E_{\pm} could be considered as the dynamic variables (that are proportional to the “true” canonical variables, actions).

In the subcritical region, $|J| < J_{\text{crit}}$, Eq. (9) can be rewritten as follows:

$$\begin{aligned} \frac{dc_{\pm}}{dt} = & -\gamma_{\text{AFM}} \left(1 \pm \frac{J}{J_{\text{crit}}}\right) c_{\pm} - \gamma \frac{\gamma_{\text{AFM}}}{\Omega_{\text{AFMR}}} c_{1,2} h_z \\ & + 2\gamma \frac{\gamma_{\text{AFM}}}{\Omega_{\text{AFMR}}} (h_x \cos \Omega_{\text{AFMR}} t + h_y \sin \Omega_{\text{AFMR}} t) \\ & + \gamma (h_x \sin \Omega_{\text{AFMR}} t - h_y \cos \Omega_{\text{AFMR}} t). \end{aligned} \quad (10)$$

It should be stressed that the same equation could be obtained directly from (6) after transition to amplitude-phase representation.

As it is seen from Eq. (10), the sign \pm corresponds to different modes which interact with the current in different ways. If $J > 0$, the effective damping of the first mode (with the amplitude c_+) increases and that of the second (with the amplitude c_-) decreases, due to the action of spin-polarised current.

Analysis of Eq. (10) shows that one component of the random magnetic field \mathbf{h} , namely, that, which is perpendicular to the plane of \mathbf{L} rotation (h_z in our notations), is a source of multiplicative noise. However, if the damping is rather small, $\gamma_{\text{AFM}} \ll \Omega_{\text{AFMR}}$, the term with multiplicative noise can be omitted. To this end Eq. (10) generates the following Fokker-Planck equations:

$$\frac{\partial f(c_{\pm})}{\partial t} = \frac{\partial}{\partial c_{\pm}} \left[\gamma_{\text{AFM}} \left(1 \pm \frac{J}{J_{\text{crit}}}\right) c_{\pm} f(c_{\pm}) + D \gamma^2 \frac{\partial f(c_{\pm})}{\partial c_{\pm}} \right]. \quad (11)$$

From the stationary solution of (11) one gets the AFM probability distribution function $f(E_+, E_-)$:

$$f(E_+, E_-) = f_0 \exp \left\{ -\frac{\gamma_{\text{AFM}} H_E}{D \Omega_{\text{AFMR}}^2 M_0} \left[\left(1 + \frac{J}{J_{\text{crit}}}\right) E_+ + \left(1 - \frac{J}{J_{\text{crit}}}\right) E_- \right] \right\}, \quad (12)$$

where f_0 is a normalization constant.

In the absence of current the distribution (12) should coincide with the Boltzmann distribution function. From the fluctuation-dissipation theorem we get the diffusion coefficient for AFM particle

$$D_{\text{AFM}} = \frac{\gamma_{\text{AFM}} H_E}{\Omega_{\text{AFMR}}^2 M_0} T = \frac{1}{\gamma M_0} \frac{\gamma_{\text{AFM}}}{\Omega_{\text{AFMR}}} \sqrt{\frac{H_E}{H_a}} T, \quad (13)$$

where $H_a \equiv \Omega_{\text{AFMR}}^2 / (\gamma^2 H_E)$ is the field of magnetic anisotropy²⁴ which in the typical AFMs is small compared with strong exchange field: $H_a \ll H_E$.

Remind, that the analogous coefficient for FM particle has a form³⁴:

$$D_{\text{FM}} = \frac{1}{\gamma M_0} \frac{\gamma_{\text{FM}}}{\Omega_{\text{FMR}}} T, \quad (14)$$

where we used an explicit expression for the Gilbert damping parameter through the frequency and half-width of FMR, $\alpha_G \equiv \gamma_{\text{FM}} / \Omega_{\text{FMR}}$. Comparing (13) and (14) one can easily see that the diffusion coefficient in AFMs is greater than that for FMs due to the large factor $\sqrt{H_E / H_a} \gg 1$, other things being equal. This is one more manifestation of the above mentioned exchange enhancement peculiar to AFM materials.

In the presence of spin-polarised current the distributions (12) are still Boltzmann-like with two (instead of one for FMs) different effective temperatures for each mode:

$$T_{\text{eff}}^{\pm} = \frac{T}{1 \pm J/J_{\text{crit}}}. \quad (15)$$

Expression (15) shows that the temperature of the “soft” mode (that one which becomes unstable at $J \rightarrow J_{\text{crit}}$) crucially growth, while the temperature of the other mode diminishes. This fact illustrates the current-induced energy swap between two modes. Seeming singularity at $J \rightarrow J_{\text{crit}}$ is an artifact of approximation which presupposes existence of high energy barrier between the different stable states.

In the supercritical region one can get the distribution function in a similar way. Neglecting, whenever it is possible, the small value $\gamma_{\text{AFM}} / \Omega_{\text{AFMR}} \ll 1$, we arrive at the following expression:

$$f(E_+, E_-) = f_0 \exp \left[-\frac{4E_+}{[3 + 4(J/J_{\text{crit}})^2]T} - \frac{(E_- - E_-^{(0)})^2}{2TE_-^{(0)}} \right], \quad (16)$$

where E_+ is related with oscillations of AFM vector in z direction and the average energy of the second mode (related with the rotation of AFM vector in xy plane), $E_-^{(0)} = M_0 H_a (J/J_{\text{crit}})^2$ is proportional to the current value.

Like in FMs⁶, the distribution (16) is Gaussian-like with respect to the energy of the second mode. However, in contrast to FM, the half-width of corresponding distribution is proportional to $\Delta E_- \propto J$, so the “quality factor” $E_-^{(0)}/\Delta E_- \propto J$ growth with the current value³⁵. Another peculiarity of AFM system compared with FM is the presence of the additional energy fluctuations related with the first mode.

IV. CONCLUSIONS

In the present paper we have derived the Langevin and the Fokker-Planck equations that take into account the peculiarities of the dynamics of AFM (in contrast to magnetisation) vector. These equations could be used for calculations of the dwell times between the different states of AFM particle and the linewidth of resonances induced by external fields (including spin-polarised current). It is shown that the thermal noise generated by fluctuating magnetic field is multiplicative. As a result, corresponding Fokker-Planck equation is nonlinear and this opens a possibility for noise-induced transitions and stochastic resonances in the system.

In the framework of the proposed approach we have calculated the AFM energy distribution function for the particular case of the collinear two sublattice AFM in the presence of spin-polarised current. We found that at a given temperature and quality factor of the magnetic resonance the diffusion coefficient of AFM shows an exchange enhancement compared to that of FM nanoparticle. It is also shown that spin-polarised current affects the effective temperatures of the normal oscillation modes in different ways: in the subcritical region the temperature of the soft mode increases and the temperature of the other (“hard”) mode decreases. In the supercritical region the energy fluctuations of the soft mode grow with respect to the current value slower than the average energy of the mode. This opens a way to control the efficiency of energy transfer from the current to AFM oscillator.

In our modeling we considered only the magnetic sources of noise. However, in the presence of spin-polarised current the fluctuations of the current value could be a source of multiplicative noise, as seen e.g. from Eq. (10). The problem of current-induced noise needs a special treatment that accounts for the relations between the magnetic state of AFM layer and resistivity, Joule losses etc.

Another important extension of the problems considered in the present paper is seen in analysis of the possible current-induced nonequilibrium states and their thermodynamics and information characteristics in the spirit of recent general approaches^{36,37}.

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